# Chapter 12

## Further Results

In this sub-chapter for some recently received experimental results theoretical essays are given to explain these results.

## 12.1 Anomalous Flyby

In this chapter an explanation of the anomalous Earth flyby is given. We follow along the lines of the article [$Pet$ 11$b$]. Let us consider an observer in the preferred reference frame $\Sigma'$ of the Earth. The boundary of the Earth is

$$X' = R\left(sin\left(\frac{2\pi}{T}t'\right)\cos\vartheta_0\,, -cos\left(\frac{2\pi}{T}t'\right)cos\vartheta_0, sin\vartheta_0\right) \tag{12.1}$$

where $R$ denotes the radius of the Earth and $T$ is the time of one day. The velocity of the boundary is given by

$$\frac{dX\prime}{dt\prime} = \frac{2\pi R}{T}\left(cos\left(\frac{2\pi}{T}t'\right)cos\vartheta_0, sin\left(\frac{2\pi}{T}t'\right)cos\vartheta_0, 0\right). \tag{12.2}$$

The velocity of a distant object (spacecraft) moving relative to the observer in $\Sigma'$ can be given by

$$\frac{dx\prime}{dt\prime} = |w'|(cos\varphi cos\vartheta, sin\varphi cos\vartheta, sin\vartheta) \tag{12.3}$$

where $\varphi$ and $\vartheta$ are fixed.

The motion of several space-crafts during the near Earth flyby shows an unexplained frequency shift which is interpreted as unexpected velocity change called Earth flyby anomaly. Let us now consider an observer on the boundary of the rotating Earth, i.e. in the non-preferred reference frame $\Sigma$ moving with velocity

$$v' = \frac{dX\prime}{dt\prime}. \tag{12.4}$$

The proper- time in this frame is by the use of (11.8)

$$\begin{aligned}(cd\tau)^2 &= -|dx|^2 + \left(\frac{v'}{c}, dx\right)^2 + 2\left(\frac{v'}{c}, dx\right)dct + (dct)^2 \\ &= -|dx|^2 + \left(1 + \left(\frac{v'}{c}, \frac{1}{c}\frac{dx}{dt}\right)\right)^2 (dct)^2.\end{aligned} \tag{12.5}$$

Therefore, the transformations from the preferred reference frame $\Sigma'$ of the non-rotating Earth into the preferred frame $\Sigma$ of the rotating Earth can be given by (compare (11.7))

$$dx^{i\prime} = dx^i, \quad dx^{4\prime} = dx^4\left(1 + \left(\frac{v\prime}{c}, \frac{1}{c}\frac{dx}{dt}\right)\right). \tag{12.6}$$

Let $k' = (k_1', k_2', k_3', k_4')$ be the wave four-vector of a plane wave in $\Sigma'$ then the corresponding wave four-vector $k = (k_1, k_2, k_3, k_4)$ in $\Sigma$ has by the transformation rules

$$k_i = {k_l}' \frac{\partial x^{l\prime}}{\partial x^i} \qquad \text{(i=1,2,3,4)}$$

the form

$$k_i = {k_i}' \quad \text{(i=1,2,3)}, \; k_4 = {k_4}'\left(1 + \left(\frac{v\prime}{c}, \frac{1}{c}\frac{dx}{dt}\right)\right).$$

The last relation gives for the frequency $\nu$ on the rotating Earth

$$\nu = \nu'\left(1 + \left(\frac{v\prime}{c}, \frac{1}{c}\frac{dx}{dt}\right)\right) \tag{12 7}$$

where $\nu'$ is the frequency measured in $\Sigma'$. In the frame $\Sigma'$ the well-known Doppler-frequency formula

$$\nu' = \gamma {\nu_0}'\left(1 + \left|\frac{1}{c}\frac{dx\prime}{dt\prime}\right| cos\left(v_l'; \frac{dx\prime}{dt\prime}\right)\right)$$

holds where

$$\gamma = \left(1 - \left|\frac{1}{c}\frac{dx'}{dt'}\right|^2\right)^{-1/2}$$

and $\left(v_l'; \frac{dx\prime}{dt\prime}\right)$ denoted the angle between the light-velocity ${v_l}'$ and $\frac{dx\prime}{dt\prime}$. Therefore, we have in $\Sigma$ by the use of (12.7) for the arriving frequency $\nu$ of the photon emitted by the moving object

$$\nu = \gamma {\nu_0}'\left(1 + \left|\frac{1}{c}\frac{dx\prime}{dt\prime}\right| cos\left(v_l'; \frac{dx\prime}{dt\prime}\right)\right)\left(1 + \left(\frac{v\prime}{c}, \frac{1}{c}\frac{dx}{dt}\right)\right).$$

This yields the frequency shift

$$\frac{\nu - \nu\prime}{{\nu_0}'} \approx \gamma\left(\frac{v\prime}{c}, \frac{1}{c}\frac{dx}{dt}\right). \tag{12.8}$$

Let the indices $*_a$ and $*_b$ mean after and before the perigee. Then (12.8) gives for the two-way frequency jump

$$2\frac{\Delta\nu}{\nu_0{}'} \approx 2\left\{\left(\frac{v_a{}'}{c}, \frac{1}{c}\frac{dx_a}{dt}\right) - \left(\frac{v_b{}'}{c}, \frac{1}{c}\frac{dx_b}{dt}\right)\right\} \tag{12.9}$$

Let us now assume that for distant objects

$$\left|\frac{dx_a}{dt}\right| \approx \left|\frac{dx_b}{dt}\right| \approx |w|.$$

We now apply the result (12.9) to the rotating Earth with velocity (12.4) with (12.2) and a distant object moving with velocity (12.3). It follows

$$2\frac{\Delta\nu}{\nu_0{}'} \approx \left|\frac{w}{c}\right|\frac{2\pi R}{T} cos\vartheta_0 \left\{cos\vartheta_a cos\left(\frac{2\pi}{T}t_a' - \varphi_a\right) - cos\vartheta_b cos\left(\frac{2\pi}{T}t_b{}' - \varphi_b\right)\right\} \tag{12.10}$$

where $t'$ is the time when the photon emitted at the distant object arrives at the observer. For the special case

$$\frac{2\pi}{T}t_a' = \varphi_a, \ \frac{2\pi}{T}t_b' = \varphi_b \tag{12.11}$$

formula (12.10) gives the two-way frequency jump

$$2\frac{\Delta\nu}{\nu_0} \approx \left|\frac{w}{c}\right|\frac{2\pi R}{T}\cos\vartheta_0(cos\vartheta_a - cos\vartheta_b). \tag{12.12}$$

Hence, an observer at the poles, i.e. $\vartheta_0 = \pm\frac{\pi}{2}$ does not measure a frequency jump. Furthermore, there is no frequency jump when the spacecraft moves symmetrically about the plane of the equator, i.e. $\vartheta_a = -\vartheta_b$.

It is an open question whether the formula (12.12) or the more general formula (2.10) may explain the anomalous flyby of all the different spacecrafts.

It is worth to mention that the measured frequency jump doesn't imply a jump of the velocity of the spacecraft passing near the Earth. This may be the reason for the difficulty to explain the anomalous Earth flyby. The idea that the rotation of the Earth may explain the flyby anomaly is at first stated by Anderson [$And$ 08]. It is worth to mention that the formula in [$And$ 08] agrees with formula (12.12) for $\vartheta_0 = 0$, i.e. the observer is on the equator.

A great value of the flyby anomaly is measured by the spacecraft NEAR. The spacecraft ROSETTA showed in the years 2007 and 2009 no flyby anomaly (see e.g. http) whereas the formula in [$And$ 08] predicts a small jump. It may be that formula (12.12) implies a very small frequency jump of ROSETTA if the observer has a suitable declination $\vartheta_0$. Mbelek [$Mbe$ 08 ] also studies the rotating Earth by the use of the transverse Doppler effect of special relativity and by some non-standard considerations.

General remarks about the problem of explaining the flyby anomaly can be found in [$http$].

## 12.2 Equations of Maxwell in a Medium

We consider a reference frame for which the pseudo-Euclidean geometry holds. The equations of Maxwell in empty space have a simple form and are derived from a Lagrangian. In a medium magnetic permeability and electric permittivity exist. The equations of Maxwell in a medium are also well-known but they cannot be derived as in empty space.

In addition to the pseudo-Euclidean metric a tensor of rank two is stated with which the proper-time in a medium is defined. The theory of Maxwell now follows along the lines of empty space. We follow the article of Petry [$Pet$ 10$b$]. Similar considerations can be found in the book of Hehl et al. [$Heh$ 03] in the chapter about the metric by an alternative method.

We start from the pseudo-Euclidean metric (1.1), (1.4) and (1.5). The equations of Maxwell in a medium are well-known and are stated in many textbooks. They have the form

$$rot\ H - \frac{1}{c}\frac{\partial D}{\partial t} = \frac{4\pi}{c}J,\ div\ D = 4\pi\rho, \tag{12.13a}$$

$$rot\ E + \frac{1}{c}\frac{\partial B}{\partial t} = 0,\ div\ B = 0 \tag{12.13b}$$

with the electric current density $J = (J^1, J^2, J^3)$ and the electric charge $\rho$. We assume a simple medium with electric permittivity $\varepsilon$ and magnetic permeability $\mu$. The connection between electric and magnetic fields $E$ and $B$ and the derived fields $D$ and $B$ is given by

$$D = \varepsilon E,\ \ B = \mu H. \tag{12.14}$$

The absolute value of light-velocity in a medium is

$$|v_l| = c/\sqrt{\varepsilon\mu} = \frac{c}{n} \tag{12.15}$$

where $n$ is the refraction index of the medium.

We now define in analogy to the theory of gravitation in flat space-time in addition to the metric the tensors

$$(g_{ij}) = \sqrt{\mu} diag\left(1,1,1,-\frac{1}{\varepsilon\mu}\right) \tag{12.16a}$$

with the inverse tensor

$$(g^{ij}) = \frac{1}{\sqrt{\mu}} diag(1,1,1,-\varepsilon\mu). \tag{12.16b}$$

The proper-time in the medium is given by

$$(cd\tau)^2 = -g_{kl}dx^k dx^l = -\sqrt{\mu}\left(|dx|^2 - \frac{1}{\epsilon\mu}(dct)^2\right). \tag{12.17}$$

We get from (12.17) by the use of $d\tau = 0$ the light-velocity (12.15).

Let $A_i$ (i=1,2,3,4) be the electro-magnetic potentials and define the anti-symmetric tensors

$$F_{ij} = \frac{\partial A_j}{\partial x^i} - \frac{\partial A_i}{\partial x^j}. \tag{12.18a}$$

Furthermore, we define the tensors

$$F^{ij} = g^{ik} g^{jl} F_{kl} \tag{12.18b}$$

and let $J = (J^1, J^2, J^3, J^4)$ be the electric four-current density. We consider the covariant differential equations

$$\frac{\partial}{\partial x^k} F^{ki} = \frac{4\pi}{c} J^i \quad \text{(i=1,2,3,4)} \tag{12.19a}$$

$$\frac{\partial}{\partial x^k} F_{ij} + \frac{\partial}{\partial x^i} F_{jk} + \frac{\partial}{\partial x^j} F_{ki} = 0 \quad \text{(i,j,k=1,2,3,4).} \tag{12.19b}$$

It immediately follows by the definition (12.18a) that the equations (12.19b) are fulfilled.

The electric field $E$ and the magnetic field $B$ are defined by

$$E = (F_{41}, F_{42}, F_{43}), \quad B = (F_{32}, F_{13}, F_{21}) \tag{12.20a}$$

then, the differential equations (12.19b) are identical with the equations (12.13b) of Maxwell.

Put for the derived fields

$$H=(F^{32},F^{13},F^{21}),\ \ D=(F^{14},F^{24},F^{34}). \tag{12.20b}$$

Then, the differential equations (12.19a) are identical with the equations (12.13a) of Maxwell. It follows by (12.20) and (12,16)

$$H=\frac{1}{\mu}B,\ \ \ D=\varepsilon E. \tag{12.21}$$

Hence, we have received a reformulation of the equations of Maxwell in a medium similar to the equations of Maxwell in empty space.

Since the relations (12.19b) are fulfilled by the use of (12.18a) the potentials $A_i$ must be calculated by relation (12.19a), i.e., for constant values of $\mu$ and $\varepsilon$

$$\frac{\partial}{\partial x^m}g^{mk}\left(\frac{\partial A_i}{\partial x^k}-\frac{\partial A_k}{\partial x^i}\right)=\frac{4\pi}{c}g_{ik}J^k.\ \ \text{(i=1,2,3,4)}$$

By the use of the Lorentz-gauge

$$\frac{\partial}{\partial x^m}\left(g^{mk}A_k\right)=0 \tag{12.22}$$

the relation can be rewritten in the form

$$\frac{\partial}{\partial x^m}\left(g^{mk}\frac{\partial A_i}{\partial x^k}\right)=\frac{4\pi}{c}g_{ik}J^k.\ \ \ \text{(i=1,2,3,4)} \tag{12.23}$$

Hence, we get four differential equations (12.23) with the gauge condition (12.22) for the four potentials $A_i$ (i=1,2,3,4).

The Lagrangian for the electro-magnetic field is

$$L_E=-\frac{1}{4}F^{kl}F_{kl}+\frac{4\pi}{c}A_kJ^k \tag{12.24}$$

with the definition (12.18). The energy-momentum tensor of the electro-magnetic field has the form

$$T(E)^i_j=\frac{1}{4\pi}\left(F^{ik}F_{jk}-\frac{1}{4}\delta^i_jF^{kl}F_{kl}\right). \tag{12.25a}$$

The tensor

$$T(E)^{ij}=g^{jk}T(E)^i_k \tag{12.25b}$$

is symmetric.

The equations of motion of a charged particle in the electro-magnetic field follow from the conservation law of the whole energy-momentum, i.e.

$$\frac{\partial}{\partial x^k}\left(T(E)_i^k + T(M)_i^k\right) = 0 \tag{12.26}$$

with

$$T(M)_j^i = \rho(E) g_{jk} \frac{dx^k}{d\tau}\frac{dx^i}{d\tau} \tag{12.27}$$

where $\rho(E)$ denotes the charge density.

All these results can be found in the article $[Pet\ 10b]$. The equations of Maxwell in a medium are also studied in a non-preferred reference frame (see sub-chapter 11.2).

## 12.3 Cosmological Models and the Equations of Maxwell in a Medium

We will now state a combination of electrodynamics in a medium (chapter 12.2) and the universe given by the use of absolute time $t'$ by formula (8.6). The proper time in the universe is given by (8.6). In chapter 12.2 the used time $t$ is the absolute time for the equations of Maxwell and in the universe the time $t'$ is the absolute time. Therefore, it is ingenious to use in this sub-chapter for the equations of Maxwell in the universe the absolute time $t'$.

In the following we put as combination of chapter 12.2 and of the universe for the potentials of electrodynamics in a medium and of gravitation

$$\left(g_{ij}{}'\right) = a^2(t')\sqrt{\mu}\, diag\left(1,1,1,-\frac{1}{\varepsilon\mu}\right) \tag{12.28a}$$

with the inverse tensor

$$\left(g^{ij\prime}\right) = \frac{1}{a^2}\frac{1}{\sqrt{\mu}}\, diag(1,1,1,-\varepsilon\mu). \tag{12.28b}$$

Then, the proper-time $\tau$ has the form

$$(cd\tau)^2 = -a^2\sqrt{\mu}\left(|dx|^2 - \frac{1}{\varepsilon\mu}(dct')^2\right). \tag{12.29}$$

The absolute value of light-velocity is again stated by (12.15). The metric is by the use of (8.5):

$$(ds)^2 = -(|dx|^2 - (a^2h)(dct')^2). \tag{12.30}$$

In the following, the covariant derivatives relative to the metric (12.30) are used.

Define

$$G' = det\left(g_{ij}{}'\right), \quad \eta' = det\left(\eta_{ij}{}'\right) \tag{12.31a}$$

and use the tensor $\left(g_{ij}\right)$ given by (12.16). Put

$$G = det\left(g_{ij}\right), \quad \eta = det\left(\eta_{ij}\right). \tag{12.31b}$$

Furthermore, we use the pseudo-Euclidean metric (1.1) with (1.5).

Let $A_i$ be the electro-magnetic potentials and define by the use of the covariant derivatives relative to the metric (12.30) the electro-magnetic field strength by

$$F_{ij} = A_{j/i} - A_{i/j}. \tag{12.32a}$$

It follows

$$F_{ij} = \frac{\partial A_j}{\partial x^i} - \frac{\partial A_i}{\partial x^j}. \tag{12.32b}$$

In addition, to the relations (12.32) we define a tensor $F^{ij\prime}$ (see chapter 12.2):

$$F^{ij\prime} = g^{ik\prime} g^{jl\prime} F_{kl}.$$

The covariant equations of Maxwell are given by

$$\left(\left(\frac{-G\prime}{-\eta\prime}\right)^{1/2} F^{ki\prime}\right)_{/k} = \frac{4\pi}{c}\left(\frac{-G}{-\eta}\right)^{1/2} J^i. \quad \text{(i=1,2,3,4)} \tag{12.33a}$$

In addition we have the covariant equations

$$F_{ij/k} + F_{jk/i} + F_{ki/j} = 0. \tag{12.33b}$$

The equation (12.33b) is identically fulfilled by virtue of (12.32a). We get from (12.28) and (12.16)

$$\left(g^{ij\prime}\right) = \frac{1}{a^2}\left(g^{ij}\right), \quad G' = -a^8\frac{\mu^2}{n^2}, \; G = -\frac{\mu^2}{n^2}, \; \eta' = -a^2 h, \; \eta = -1.$$

The equations of Maxwell (12.33a) can be rewritten

$$\left(\frac{\mu}{n}\frac{1}{a\sqrt{h}}g^{km}g^{in}F_{mn}\right)_{/k} = \frac{4\pi}{c}\frac{\mu}{n}J^i. \quad \text{(i=1,2,3,4)}$$

We define for i,j=1,2,3,4 the tensor (in analogy to chapter 12.2):

$$F^{ij} = g^{ik}g^{jl}F_{kl}. \tag{12.34}$$

Then, the equations of Maxwell have the form

$$\left(\frac{\mu}{n}\frac{1}{a\sqrt{h}}F^{ki}\right)_{/k} = \frac{4\pi}{c}\frac{\mu}{n}J^i. \quad \text{(i=1,2,3,4)} \tag{12.35}$$

The only non-vanishing Christoffel symbol of the metric (12.30) is

$$\Gamma^4_{44} = \frac{1}{a\sqrt{h}}\frac{d}{dx^{4\prime}}\left(a\sqrt{h}\right).$$

Therefore, the equations of Maxwell are

$$\frac{1}{a\sqrt{h}}\frac{\partial}{\partial x^k}\left(\frac{\mu}{n}g^{km}g^{in}F_{mn}\right) = \frac{4\pi}{c}\frac{\mu}{n}J^i. \quad \text{(i=1,2,3,4)} \tag{12.36}$$

The definitions (12.18) and (12.20) give again the relations (12.21). Furthermore, the equation of Maxwell (12.36) has for constant $\mu$ *and* $n$ the form

$$rot\ H - \frac{1}{c}\frac{\partial D}{\partial t'} = \frac{4\pi}{c}\left(a\sqrt{h}\vec{J}\right), \quad div\ D = \frac{4\pi}{c}\left(a\sqrt{h}\,J^4\right) \tag{12.37a}$$

where $\vec{J} = (J^1, J^2, J^3)$ and $\rho(E) = a\sqrt{h}\,J^4$.

The relations (12.33b) are rewritten in the form

$$rot\ E + \frac{1}{c}\frac{\partial B}{\partial t\prime} = 0, \quad div\ B = 0. \tag{12.37b}$$

The conservation of the streaming vector

$$J^k{}_{/k} = 0$$

has the form

$$\frac{\partial}{\partial x^k}\left(a\sqrt{h}\,J^k\right) = 0. \tag{12.38}$$

The equations (12.37) and (12.38) are the equations of Maxwell in a medium where (12.21) holds and which is contained in the universe. They are given relative to the metric (12.30).

## 12.4 Redshift of Distant Objects in a Medium

Here, we follow along the lines of article [$Pet\ 13a$]. We assume that the proper-time $\tau$ is given by (12.29) where $t'$ is the absolute time in the universe. We get by the use of (12.29) for an atom at rest which emits a photon at time $t_e{}'$

$$d\tau = a(t_e{}')\frac{\mu_e^{1/4}}{n_e}dt'. \tag{12.39}$$

Here, $n_e$ and $\mu_e$ mean the refraction index and the permittivity of the medium in which the photon is emitted.

This means that the emitted frequency at time $t_e{}'$ is given by

$$\nu(t_e{}') = a(t_e{}')\frac{\mu_e^{1/4}}{n_e}\nu_0 \tag{12.40}$$

where $\nu_0$ is the frequency emitted by the same atom at rest, at present time $t_0'$ and without medium. The photon moves to the observer. The equations of motion (1.30) imply for i=4

$$\frac{d}{dt\prime}\left(g_{4k}{}'\frac{dx^k\prime}{d\tau}\right) = \frac{1}{2}\frac{\partial g_{kl}}{\partial ct\prime}\frac{dx^k\prime}{dt\prime}\frac{dx^l\prime}{dt\prime}\frac{dt\prime}{d\tau}$$

We assume that the refraction index and the permittivity are depending on space but not on the time. This is justified by the equations of Maxwell (12.36) with the notations (12.32), (12.34), (12.16) and (12.21). Hence, we have

$$\frac{d}{dt\prime}\left(g_{44}{}'\frac{dct\prime}{d\tau}\right) = a\frac{da}{dct\prime}\mu^{1/2}\left(\left|\frac{dx\prime}{dt\prime}\right|^2 - \left(\frac{c}{n}\right)^2\right)\frac{dt\prime}{d\tau}$$

It follows by the use of (12.29)

$$\frac{d\tau}{dt\prime} = a\mu^{1/4}\frac{1}{c}\left(\left(\frac{c}{n}\right)^2 - \left|\frac{dx\prime}{dt\prime}\right|^2\right)^{1/2}$$

We get by the substitution of this relation into the above equation

$$\frac{d}{ct\prime}\left(g_{44}{}'\frac{dct\prime}{d\tau}\right) = -\mu^{1/4}\frac{da}{dt'}\left(\left(\frac{c}{n}\right)^2 - \left|\frac{dx'}{dt'}\right|^2\right)^{1/2} = 0$$

for the velocity of light, i.e. the energy of the photon during its motion is conserved. This means that the frequency is not changed by the use of the law of Planck. Hence, the frequency $\nu$ which arrives at the observer is

$$\nu = \nu(t_e{}') = a(t_e{}')\frac{\mu_e^{1/4}}{n_e}\nu_0.$$

This yields the redshift

$$z = \frac{\nu_0}{\nu} - 1 = \frac{1}{a(t_e{}')}\frac{n_e}{\mu_e^{1/4}} - 1. \tag{12.41}$$

Taylor expansion of $a(t)$ yields

$$a(t_e') = a(t_0{}') + \dot{a}(t_0{}')(t_e' - t_0{}') + \frac{1}{2}\ddot{a}(t_0{}')(t_e' - t_0{}')^2 + \cdots$$

It is assumed that the photon's path from source to receiver is only a small fractional part that is within the medium, i.e. by virtue of (8.14) it holds $t_e' - t_0' = -\frac{r}{c}$. This implies by the use of the initial conditions

$$a(t_e{}') = 1 - H_0\frac{r}{c} + \frac{1}{2}\frac{\ddot{a}(t_0{}')}{H_0^2}\left(H_0\frac{r}{c}\right)^2 + \cdots \tag{12.42a}$$

Relation (8.15) yields by the use of (7.17), (7.28) and $\Omega_r \ll 1$:

$$\frac{\ddot{a}(t_0{}')}{H_0^2} \approx 2 - \frac{3}{2}\Omega_m. \tag{12.42b}$$

The redshift (12.41) is by the use of (12.42) given in the form

$$z = \frac{n_e}{\mu_e^{1/4}} - 1 + \frac{n_e}{\mu_e^{1/4}}\left(H_0\frac{r}{c}\right) + \frac{3}{4}\Omega_m\frac{n_e}{\mu_e^{1/4}}\left(H_0\frac{r}{c}\right)^2. \tag{12.43}$$

The redshift formula (12.43) gives the whole value of the redshift. It follows partly from the universe and partly from the medium in which light is emitted. An intrinsic redshift is discussed by several authors who neglect an expanding universe. (see e.g. [$Fah$ 95] ). It is shown in the articles [$Pet$ 97], [$Pet$ 07], [$Pet$ 11$a$], [$Pet$ 13$b$] that the redshift in the universe can also be interpreted with the aid of the different kinds of energy which are transformed into one another in the course of time as stated in chapter VIII. The interpretation of an expanding space is not necessary. An extensive study of a non-expanding universe exists, too (see e.g. [$Fah$ 95], [$Ler$ 05 ],[$Alf$ 10]).

Let us assume for the refraction index $n$ and for the permittivity the representation

$$n_e = 1 + \Delta \mathrm{n}, \quad \mu_e = 1 + \Delta\mu.$$

Then, we get from (12.43)

$$z = \Delta n - \frac{1}{4}\Delta\mu + \frac{1}{4}(\Delta\mu)^2 + \frac{n_e}{\mu_e^{1/4}}\left(H_0\frac{r}{c}\right) + \frac{3}{4}\Omega_m\frac{n_e}{\mu_e^{1/4}}\left(H_0\frac{r}{c}\right)^2 + \cdots \quad (12.44)$$

Discussion:

(1) Relation (12.43) or (12.44) implies for a fixed redshift of a galaxy (quasar) in a medium that the distance to this object is in general, i.e.

$$\frac{n_e}{\mu_e^{1/4}} > 1,$$

smaller than without medium.

(2) The linear Hubble law can give an overestimate of the Hubble constant which depends also on the different media.

(3) Quasars may be nearer by the use of (12.43) or (12.44) than by the standard Hubble law. This yields that the measured energy emitted from these quasars is smaller than generally assumed.

(4) Two galaxies (quasars) in different media can give the same redshifts although the distances to these objects are different.

(5) Galaxies and quasars with nearly the same distances can have quite different redshifts which depend on the media in which light is emitted. Measurements confirm this result (see e.g., $[Arp\ 88], [Fah\ 95]$).

(6) It may be that dark energy is not necessary, i.e. $\Omega_\Lambda = 0$ because formula (12.43) or (12.44) may explain the redshifts of galaxies, of quasars, too.

Furthermore, there exists no age problem for the universe because the absolute time$t'$ must be used instead of the proper-time $\tilde{\tau}$ (see (8.19)).

## 12.5 Flat Rotation Curves in Galaxies with Media

In this chapter we assume that every object, e.g. Earth, Sun, galaxy, quasar, etc. is surrounded by a medium. Furthermore, let us omit the universe, i.e. the objects are not too far from us. In addition to the object we consider the surrounding medium. In the following, only the approximations of Newton are used. Hence, we have $a(t) = h(t) = 1$ implying by the use of the Newtonian approximation the line-element of the pseudo-Euclidean metric and the proper-time

$$(cd\tau)^2 = -\left(|dx|^2 - \left(\frac{1}{n^2} - \frac{2}{c^2}U\right)(dct)^2\right). \tag{12.45}$$

Here, the Newtonian potential is given by (2.32) with (2.31) in the exterior of the object:

$$U = \frac{kM_g}{r}. \tag{12.46a}$$

Furthermore, let us assume that the refraction index is of the form

$$n = 1 + \Delta n \tag{12.46b}$$

with

$$0 < \Delta n \ll 1. \tag{12.46c}$$

This yields the approximate proper-time

$$(cd\tau)^2 = -\left(|dx|^2 - \left(1 - 2\Delta n - \frac{2}{c^2}U\right)(dct)^2\right). \tag{12.47}$$

The equations of motion (1.30) give to the lowest order

$$\frac{d^2x^i}{dt^2} = \frac{1}{2}\frac{\partial g_{44}}{\partial x^i}c^2 \quad \text{(i=1,2,3)}$$

i.e. we get

$$\frac{d^2x^i}{dt^2} = \frac{\partial \Delta n c^2}{\partial x^i} + \frac{\partial U}{\partial x^i} \quad \text{(i=1,2,3).} \tag{12.48}$$

Standard methods yield the result

$$\frac{1}{2}\frac{d}{dt}\left|\frac{dx}{dt}\right|^2 = \frac{d}{dt}(\Delta n c^2 + U). \tag{12.49}$$

Furthermore, we get an anomalous acceleration into the radial direction

$$b(r) = \frac{\partial \Delta n c^2}{\partial r}. \tag{12.50a}$$

We will now assume the simple form

$$\Delta n = b_0\left(1 - \frac{3}{2}\frac{r}{r_0} + \frac{1}{2}\left(\frac{r}{r_0}\right)^2\right) \tag{12.50b}$$

with $0 < b_0 \ll 1$ and where $r_0$ is the boundary of the medium. It is assumed that the boundary of the medium $r_0$ is great compared to the boundary of the body. A solution of (12.49) is given by

$$\left|\frac{dx}{dt}\right|^2 = 2\Delta n c^2 + 2U \tag{12.51}$$

where the constant of integration is set equal to zero. This result gives the rotation curves, as e.g. of galaxies, of stars etc. The derivation of this result doesn't correspond to the usual one of rotation curves but it has regard to the refraction index.

The equations (12.51), (12.50) and (12.46) give the rotation curves and the anomalous acceleration

$$\left|\frac{dx}{dt}\right| = \left(2b_0 c^2 \left(1 - \frac{3}{2}\frac{r}{r_0} + \frac{1}{2}\left(\frac{r}{r_0}\right)^2\right) + 2U\right)^{1/2} \tag{12.52a}$$

and

$$b(r) = -\frac{b_0 c^2}{r_0}\left(\frac{3}{2} - \frac{r}{r_0}\right). \tag{12.52b}$$

It is worth to mention that for $r \ll r_0$ equation (12.52a) gives the well-known flat rotation curves.

The results (12.52) are applied to the Sun system and to galaxies:

(1) Sun system: We consider the Pioneers which give an anomalous acceleration (see e.g. [$And$ 02]):

$$a_p \approx 8.74 \cdot 10^{-8}\,\frac{cm}{s^2} \tag{12.53}$$

into the direction to the Sun. There is an extensive study of the anomalous acceleration which is confirmed by several authors. Recently, Turyshev et al. [$Tur$ 11] measured a decrease of the anomalous acceleration. This supports the explanation of an anisotropic emission of on-board heat which is also discussed by [$And$ 02] and by many otherauthors.

Relation (12.52b) gives an anomalous acceleration to the Sun which implies by the use of (12.53) with $r \approx \frac{1}{2} r_0$ ($r_0$ in cm):

$$b_0 \approx 8.74 \cdot 10^{-8}\,\frac{r_0}{c^2} \approx 0.9 \cdot 10^{-28} r_0. \tag{12.54}$$

The result (12.52b) with (12.54) gives an anomalous acceleration which also decreases with the distance from the centre of the Sun. Hence, we have a quite different interpretation of the Pioneer anomaly without an anisotropic emission of on-board heat although the anisotropic emission is the presently accepted interpretation.

There are no flat rotation curves for the planets moving around the Sun by a suitable boundary $r_0$ of the medium. This follows by formula (12.52a) with (12.54).

(2) Galaxies:Many galaxies show flat rotation curves (see e.g. [$San$ 86]). This result was already observed by Zwicky. Many authors assume dark matter to explain this result. But there are also other alternatives to explain the flat rotation curves. Milgrom [$Mil$ 83] suggests a modified Newton law.

In this chapter the rotation curves are given by the use of (12.52a), i.e.

$$\left|\frac{dx}{dt}\right| = \left(2b_0c^2\left(1 - \frac{3}{2}\frac{r}{r_0} + \frac{1}{2}\left(\frac{r}{r_0}\right)^2\right) + 2U\right)^{1/2}. \tag{12.55}$$

Here, the last expression under the square root yields the well-known rotation curves. Let us assume that $r_0$ is very large then the condition

$$b_0c^2 > U = \frac{kM_g}{r} \tag{12.56}$$

gives flat rotation curves. The luminal mass of many galaxies is

$$M \approx \alpha 10^{10} M_{\odot} \approx 2\alpha 10^{43}\ g \tag{12.57}$$

with a suitable constant $\alpha$ depending on the galaxy.

The observed distance $r$ where flat rotation curves arise is given by

$$r \geq \tilde{r} \cdot 10^{22}\ cm \tag{12.58}$$

with a suitable constant $\tilde{r}$. Relation (12.56) yields for flat rotation curves the condition

$$b_0 \geq 1.5\frac{\alpha}{\tilde{r}}10^{-7}. \tag{12.59}$$

This implies by the use of (12.52a) flat rotation curves with velocity

$$|v| \geq c \cdot 10^{-3}\sqrt{30\frac{\alpha}{\tilde{r}}}. \tag{12.60}$$

The inequality (12.60) gives the correct order of the velocity of flat rotation curves of galaxies. But every galaxy must be studied separately in detail.

Hence, the results about flat rotation curves of galaxies imply by the use of surrounding media the correct order of the measured velocities. Surrounding media of galaxies may therefore explain flat rotation curves without the assumption of dark matter. Contrary, all the dark matter contained in galaxies is not enough to explain all the dark matter in the universe. Therefore, media give the possibility to explain the results without the assumption of dark matter. Hence, we may ask whether media can be interpreted as the assumed dark matter.

Let us compute the density of dark matter produced by the reflection index. The law of Newton

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Delta nc^2}{dr}\right) = -4\pi k\rho_d \tag{12.61}$$

where $\rho_d$ denotes the density of the assumed dark matter implied by the refraction index (12.50b). We get from formula (12.61):

$$\rho_d = 3\frac{bc^2}{4\pi k}\frac{1}{r_0}\frac{1}{r}\left(1-\frac{r}{r_0}\right). \tag{12.62}$$

The boundary of the dark matter is the radius $r_0$ given by (12.50b).The mass of the dark matter is by the use of (12.62)

$$M_d = \frac{4\pi}{3}\int r^2\rho_d\,dr = \frac{b_0c^2}{6k}r_0. \tag{12.63}$$

Hence, it follows by the use of (12.54) that the assumed dark mass of the surrounding Sun is small compared to the mass of the Sun. But the dark mass of surrounding galaxies is for sufficiently large radius $r_0$ much greater than the luminous mass of the galaxy by virtue of (12.57), (12.58) and (12.59).

The density (12.62) of the assumed dark matter would give a singularity in the centre of the body but it is worth to mention that the medium surrounds the body and it is not the assumed dark matter.