Chapter 8

Handling Research Data with Inferential Statistics

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Handling Research Data with Inferential Statistics

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Abstract

Inferential statistics just like descriptive statistics has various tools under it which have their peculiar uses as regard data processing. Inferential statistical tools are used for making inferences from samples about the population from which they are selected. It is the field of statistics that has statistical tools for estimating, predicting and making decisions about population and testing hypotheses. The analytical tools within this field of statistics are not well understood and also applied by some researchers, would-be researchers as well as students. This chapter explains the various statistical tools thoroughly.

Keywords

Inferential, Research, Estimation, Sample, Population, Analysis

8.1 Introduction

With descriptive statistics it is required that all the variables considered are known before a dataset can be described with a descriptive tool. However in the case of inferential statistics, not all the variables are known. Inferential statistics involves taking a representative sample from a population to make inferences from it about the sample because it would be difficult to study all members or elements of a population. Inferential statistics is a statistical method used to test hypotheses that relate to the relationship between two variables. Descriptive statistics can be used to describe the relationships based on the data pattern or trend but inferential statistics provide a more rigorous prove as to whether there exist a relationship between two variables. Inferential statistics are used in the case of hypothesis testing and tests such as z-test, t-test, Analysis of Variance (ANOVA) and chi-square test.

8.2 T-test

It is used to test the difference between two groups that are continuous variables. For instance, testing the differences between the weights of individuals based on their ages i.e. testing whether there is weight decrease or increase as an individual ages. The equation for the t-test to use depends on whether the researcher is doing an independent samples t-test – comparing two different groups; dependent samples t-test (paired t-test) – comparing two same groups on two different periods of time or different groups matched on an important variable. There is also the one sample t-test, which is used when the researcher wants to test the group scores of a population with a known mean. It must also be stated that the equation used would also vary when doing the independent sample t-test based on the whether the two groups have the same sample size or not. The t-test is used to handle a sample size of 30 and below which is usually referred to as small sample size. The t-test is preferred when the sample size is 30 and the standard deviation of the population is unknown.

The equation for a single (one) sample t-test is given as:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

where $\bar{x}$ *=sample mean of the group,* μ *=population mean of the group, s=sample standard deviation, n=sample size, t=the t statistics.*

Let us take for instance a case where a researcher collects the test scores obtained by 25 students out of 100 marks for a class size of 30 students summarized below:

Table 8.1.1 *Scores of Students in a Class.*

61.5	54.3	1	28	2
15.1	28	33	60	40
12.5	69	19	31	58
50	25	91	67	75
1.5	15	40.4	33	81

Assuming the population mean mark for the class is 35, the t-test can be used to test where there sample mean is different from the population mean. In another scenario, a given mean score of 55, one can test whether there is a difference between this mean score and the sample mean. The t-test is used to test whether the mean of a sample differ from a known or a given mean.

From the data given the mean of the sample can be calculated using the formula:

$$\bar{x} = \frac{\sum x_1 + x_2 + x_3 + \ \ldots.. + x_n}{n}$$

$$\bar{x} = \frac{\sum 61.5 + 54.3 + 1 + \ .....+81}{25} = 39.65$$

$$where\ \bar{x} = sample\ mean$$

The sample standard deviation is given by the formula:

$$S = \sqrt{\frac{\sum(\mathrm{x} - \bar{x})^2}{n}}$$

$$S = \sqrt{\frac{\sum(\mathrm{x} - \bar{x})^2}{n}} = \sqrt{\frac{\sum(\mathrm{x}_1 - \bar{x})^2 + (\mathrm{x}_2 - \bar{x})^2 + (\mathrm{x}_3 - \bar{x})^2 + \cdots + (\mathrm{x}_\mathrm{n} - \bar{x})^2}{n}}$$

$$S = \sqrt{\frac{\sum(61.5 - 39.65)^2 + (54.3 - 39.65)^2 + (1 - 39.65)^2 + \cdots + (81 - 39.65)^2}{25}}$$

$$= 25.94$$

where n=sample size s=sample standard deviation.

Therefore

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{39.65 - 35}{\frac{25.94}{\sqrt{25}}}$$

$$t_{cal} = \frac{39.65 - 35}{\frac{25.94}{\sqrt{25}}} = \frac{4.65}{\frac{25.94}{5}} = \frac{4.65}{5.19} = 0.90$$

Thus t-calculated $(t_{cal}) = 0.90$

$$where\ \mu = population\ mean$$

Now the degree of freedom (df) of the sample which is given by the total number of sample minus one (1) i.e. $df = n - 1 = 25 - 1 = 24$. Once this is done, the degree freedom (df) obtained (24) can be used with the selected level of significance chosen either 5% (0.05) or 1% (0.01) to read the t-critical value or tabulated value. The t-calculated and t-critical values are then compared and the decisions taken as follow:

if $t_{cal} > t_{critical\ or\ tabulated\ (at\ df=24, \alpha=0.05\ or\ 0.01}$.

Then the sample mean is statistically significant from the hypothesized mean or assumed population mean.

However if $t_{cal} < t_{critical\ or\ tabulated\ (at\ df=24, \alpha=0.05\ or\ 0.01}$.

Then the sample mean is not statistically significant from the hypothesized mean or assumed population mean.

A demonstration of how to read the t-critical value from the t-test table in figures 8.1.1 and 8.1.2

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**
df							
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052

Figure 8.1.1 *T-table for 0.05 Level of Significance.*

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02
df								
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479

Figure 8.1.2 *T-table for 0.01 Level of Significance.*

Decision:

$$t_{cal}(1.71) > t_{critical\ or\ tabulated\ at\ df=24, \alpha=0.05}(0.90)$$

$$t_{cal}(2.49) > t_{critical\ or\ tabulated\ at\ df=24, \alpha=0.05}(0.90)$$

Conclusion:

Since t_{cal} is greater than $t_{crit\ at\ df=24}$ at both 5% and 1% levels of significance, then the sample mean is significantly different from the population mean or the hypothesize mark.

Using the independent t-test

The independent t-test is used to test whether the means of two independent groups are the same or differ from each other. For this test to be used the two groups must be independent from each other. The formula for the computation of the t-calculated value is given by the formula:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{ss_1 + ss_2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{\sum x_1^2 - \frac{\sum(x_1)^2}{n_1} + \sum x_2^2 - \frac{\sum(x_2)^2}{n_2}}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where SS_1=Sum of Squares of Group one, SS_2=Sum of Squares of Group two, where n_1 and n_2 are sample sizes of group one and two respectively.

It should be noted that the sample sizes may be equal or unequal depending on the data being collected.

Now let us assume a researcher want to test whether the sample means of the performance of two groups of students pursuing two different programmes based on the data summarized below:

1. *Equal Sample Size*

Table 8.2.1 *Data to Illustrate Equal Sample Size.*

Sample No.	A	B
1	25	34
2	45	59
3	63	73
3	21	43
4	19	17
5	78	47

For the table above the sample sizes for the two independent groups (A and B) are equal. Each group has a sample size of 6.

In order to compute the t-calculated value, the sample means and the sum of squares for each of group is determined.

The means of each group is calculated as follows:

$$\bar{x}_A = \frac{\sum x_1 + x_2 + x_3 + \ .....+ x_n}{n_A}$$

$$\bar{x}_A = \frac{\sum 25 + 45 + 63 + \ .....+ 78}{6} = 41.83$$

$$\bar{x}_B = \frac{\sum x_1 + x_2 + x_3 + \ .....+ x_n}{n_B}$$

$$\bar{x}_B = \frac{\sum 34 + 59 + 73 + \ .....+ 47}{6} = 45.5$$

The Sum of Square for each of the groups is also determined as follows:

$$SS_A = \sum x_A^2 - \frac{\sum (x_A)^2}{n_A}$$

$$SS_A = \sum (25^2 + 45^2 + 63^2 + \ldots + 78^2) - \frac{\sum (25 + 45 + 63 + \cdots + 78)^2}{6}$$

$$SS_A = 13505 - 10500.17 = 3004.83$$

$$SS_B = \sum x_B^2 - \frac{\sum (x_B)^2}{n_B}$$

$$SS_B = \sum (34^2 + 59^2 + 73^2 + \ldots + 47^2) - \frac{\sum (34 + 59 + 73 + \cdots + 47)^2}{6}$$

$$SS_B = 14313 - 12421.5 = 1891.5$$

$$t_{cal} = \frac{\bar{x}_A - \bar{x}_B}{\sqrt{\left(\frac{\sum x_A^2 - \frac{\sum (x_A)^2}{n_A} + \sum x_B^2 - \frac{\sum (x_B)^2}{n_B}}{n_A + n_B - 2}\right)\left(\frac{1}{n_A} + \frac{1}{n_B}\right)}}$$

$$t_{cal} = \frac{41.83 - 45.5}{\sqrt{\left(\frac{3004.83 + 1891.5}{6 + 6 - 2}\right)\left(\frac{1}{6} + \frac{1}{6}\right)}}$$

$$t_{cal} = \frac{-3.67}{\sqrt{\left(\frac{(4896.33)}{10}\right)\left(\frac{2}{6}\right)}} = \frac{-3.67}{\sqrt{489.97}} = \frac{-3.67}{22.14} = -0.17$$

The degree of freedom $(df) = (n + n - 2) = (6 + 6 - 2) = 10.$

The t-critical value can now be read from the t-test table and then compared to the t-calculated to take the decision.

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**
df							
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201

Figure 8.2.1 *T-table for 0.05 Level of Significance.*

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**
df								
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718

Figure 8.2.2 *T-table for 0.01 Level of Significance.*

Decision:

$$t_{cal}(2.23) > t_{critical\ or\ tabulated\ at\ df=10, \alpha=0.05}(-0.17)$$

$$t_{cal}(2.76) > t_{critical\ or\ tabulated\ at\ df=10, \alpha=0.01}(-0.17)$$

Conclusion:

Since t_{cal} *is greater than* $t_{crit\ at\ df=10}$ *at both 5% and 1% levels of significance, then the sample mean of the two independent group is significantly different.*

2. *Unequal Sample Size*

Table 8.3.1 *Data to Illustrate Equal Sample Size.*

Sample No.	A	B
1	25	34
2	45	59
3	63	73
4	21	43
5	19	
6	78	

In the case given above, the two independent groups have unequal sample sizes: Group has a sample size of 6 and B, a sample size of 4. This case can be handled in a similar way as done below: The means of each group is calculated as follows:

$$\bar{x}_A = \frac{\sum x_1 + x_2 + x_3 + \ \ldots\ldots + x_n}{n_A}$$

$$\bar{x}_A = \frac{\sum 25 + 45 + 63 + \ \ldots\ldots + 78}{6} = 41.83$$

$$\bar{x}_B = \frac{\sum x_1 + x_2 + x_3 + \ \ldots\ldots + x_n}{n_B}$$

$$\bar{x}_B = \frac{\sum 34 + 59 + 73 + \ 43}{4} = 52.25$$

The Sum of Square for each of the groups is also determined as follows:

$$SS_A = \sum x_A^2 - \frac{\sum (x_A)^2}{n_A}$$

$$SS_A = \sum(25^2 + 45^2 + 63^2 + \ldots + 78^2) - \frac{\sum(25 + 45 + 63 + \cdots + 78)^2}{6}$$

$$SS_A = 13505 - 10500.17 = 3004.83$$

$$SS_B = \sum x_B^2 - \frac{\sum(x_B)^2}{n_B}$$

$$SS_B = \sum(34^2 + 59^2 + 73^2 + 43^2) - \frac{\sum(34 + 59 + 73 + 43)^2}{4}$$

$$SS_B = 11815 - 10920.25 = 894.75$$

$$t_{cal} = \frac{\bar{x}_A - \bar{x}_B}{\sqrt{\left(\frac{\sum x_A^2 - \frac{\sum(x_A)^2}{n_A} + \sum x_B^2 - \frac{\sum(x_B)^2}{n_B}}{n_A + n_B - 2}\right)\left(\frac{1}{n_A} + \frac{1}{n_B}\right)}}$$

$$t_{cal} = \frac{41.83 - 52.25}{\sqrt{\left(\frac{3004.83 + 894.75}{6 + 4 - 2}\right)\left(\frac{1}{6} + \frac{1}{4}\right)}}$$

$$t_{cal} = \frac{-3.67}{\sqrt{\left(\frac{(3899.58)}{8}\right)\left(\frac{5}{12}\right)}} = \frac{-3.67}{\sqrt{487.86}} = \frac{-3.67}{22.09} = -0.17$$

The degree of freedom$(df) = n + n - 2 = (6 + 4 - 2) = 8$

Now the researcher can take the decision by reading t-critical value from the t-test table and then compared to the t-calculated value.

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	~~0.000~~	~~0.706~~	~~0.889~~	~~1.108~~	~~1.397~~	~~1.860~~	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781

Figure 8.3.1 *T-table for 0.05 Level of Significance.*

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02
df								
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998
8	~~0.000~~	~~0.706~~	~~0.889~~	~~1.108~~	~~1.397~~	~~1.860~~	~~2.306~~	2.896
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718

Figure 8.3.2 *T-table for 0.01 Level of Significance.*

Decision:

$$t_{cal}(2.31) > t_{critical\ or\ tabulated\ at\ df=8,\alpha=0.05}(-0.17)$$

$$t_{cal}(2.90) > t_{critical\ or\ tabulated\ at\ df=8,\alpha=0.01}(-0.17)$$

Conclusion:

Since t_{cal} is greater than $t_{crit\ at\ df=10}$ at both 5% and 1% levels of significance, then the sample mean of the two independent group is significantly different.

Z-test

The Z-test also functions like the t-test but the difference here is that when the sample size exceeds 30 and the standard deviation of the population is known, the data is considered as a large sample and thus the Z-test becomes the appropriate test. There are several Z-tests just like the t-tests. These include: one sample Z-test for proportions; two sample z-Test for Proportions; one sample Z-test; two sample Z-test with equal variance; two sample Z-test with unequal variance.

One sample Z-test for proportions

This is used to test whether the proportions of a sample are the same or different. Let us say there is a claim made by buyers that 5 out of 10 cars manufactured by a company are faulty. To test this claim by the buyers assuming a random sample of 35 cars manufactured by the company out of 60 were faulty; the one sample test for proportions can be used.

To test this claim, the one sample test for proportions is used by following the procedure below:

State the hypothesis

$$P = \frac{S}{n} = \frac{9}{10} = 0.9$$

$$\hat{P} = \frac{s_T}{n_T} = \frac{35}{60} = 0.58$$

where p=proportion n=number of proportion, s_T=Scores or counts out of random sample, nT=Total random sample number, s=scores or counts out of number of proportion.

$$H_o: P = 0.5$$

$$H_1: P \neq 0.5$$

Choose the level of significance

Let us use $\alpha = 5\ or\ 0.05$.

State the decision rule:

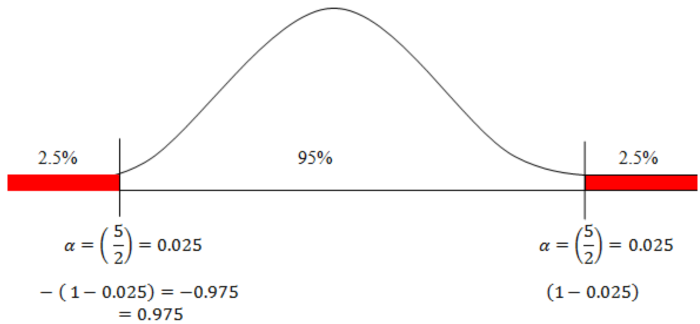

Figure 8.4.1 *Illustration how the chosen* $\boldsymbol{\alpha}$ *should be handled.*

Since the hypothesis is non directional i.e. (the equal sign has been used). It means it is a two tailed tests, therefore we use $\alpha = \frac{5}{2} = 0.025$, in the right and left directions, hence to find the Z-value in the left direction look for the Z-value for $(1 - 0.025) = 0.975$ in both directions. The Z- value for 0.0975 in the table is -1.96 and 1.96.

$$Reject\ H_0\ if\ Z < -1.96\ or\ Z > 1.96$$

$$Fail\ to\ reject\ H_0\ if\ -1.96 \leq Z \leq 1.96$$

Calculate the test statistics (Z- calculated)

$$Z = \frac{\hat{P} - P}{\sqrt{\frac{P(1-P)}{n_T}}}$$

$$Z = \frac{0.58 - 0.90}{\sqrt{\frac{0.90(1-0.90)}{100}}}$$

$$Z = \frac{-0.32}{\sqrt{\frac{0.90(0.1)}{100}}} = \frac{-0.32}{\sqrt{0.0009}} = \frac{-0.32}{0.03} = -10.67$$

State the results

$$Z = -10.67$$

$$Reject\ H_0\ because\ Z < -1.96$$

Make the conclusion

It can therefore be concluded there that the claim made by the buyers that 5 out of 10 manufactured cars are faulty is true.

Two sample Z-test for proportions

The two sample proportion test is used to test whether proportions of two samples are the same or different. If there is a claim that drug A is more efficacious than drug B, and a random sample of 40 out of 100 patients and 56 out of 100 patients recovered from the same disease as a result of administering drugs A and B respectively to them.

State the hypothesis

$$P_1 = \frac{s_1}{n_1} = \frac{44}{100} = 0.40$$

$$P_2 = \frac{s_2}{n_2} = \frac{56}{100} = 0.56$$

$$\hat{P} = P_1 + P_2 = \frac{40}{100} + \frac{56}{100} = 0.40 + 0.56 = 0.96$$

where P_1=proportion 1, P_2=proportion 2, n_1=number of proportion 1, n_2=number of proportion 2, s_1=scores or counts out for proportion 1, s_2=scores or counts out for proportion 2

$$H_o: P_1 = P_2$$

$$H_1: P_1 \neq P_2$$

Choose the level of significance

Let us choose $\propto = 0.05\ 0r\ 5\%$

State the decision rule

$$Reject\ H_0\ if\ Z < -1.96\ or\ Z > 1.96$$

$$Fail\ to\ reject\ H_0\ if\ -1.96 \leq Z \leq 1.96$$

Calculate the test statistics (Z- calculated)

$$Z = \frac{P_1 - P_2}{\sqrt{\hat{P}\left(1-\hat{P}\right)}\ x\ \sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

$$Z = \frac{0.40 - 0.56}{\sqrt{0.96(1-0.96)}\ x\ \sqrt{\frac{1}{100}+\frac{1}{100}}}$$

$$Z = \frac{-0.16}{\sqrt{0.96(0.04)}\ x\ \sqrt{\frac{2}{200}}}$$

$$Z = \frac{-0.16}{\sqrt{0.04}\ x\ \sqrt{0.01}} = \frac{-0.16}{0.2\ x\ 0.1} = -8.0$$

State the results

$$Z = -8.0$$

$$Reject\ H_0\ because\ Z < -1.96$$

Make the conclusion

The null hypothesis therefore must be rejected on the grounds that the data does not support it hence the alternate hypothesis is true i.e. drug A is more efficacious compared to drug B.

One sample Z-test

It is used for testing the claims when data is collected on two samples with equal variance. For instance, let us consider a case where the weights of mean in a country are normally distributed with a mean of 67kg and standard deviation of 6. Data on the weight of a sample size of 40 men within a city in that country was collected, and the mean weight determined to be 65Kg. One can then test whether the mean weight of men in the city is higher than that of the country. To test this, the following procedures are followed:

State the hypothesis

$$H_o: \bar{x} = \mu$$

$$H_1: \bar{x}_1 \neq \mu$$

Choose the level of significance

$$Let \ \propto = 0.05 \ or \ 5\%$$

State the decision rule

$$Reject\ H_0\ if\ Z < -1.96\ or\ Z > 1.96$$

$$Fail\ to\ reject\ H_0\ if\ -1.96 \le Z \le 1.96$$

Calculate the test statistics (Z-calculated)

$$Z = \frac{\bar{x} - \mu}{\sqrt{\frac{\delta}{n}}}$$

$$\bar{x} = 65\ \mu = 67 = 70\ \delta = 6\ n = 40$$

$$Z = \frac{65 - 67}{\sqrt{\frac{6}{40}}} = \frac{-2}{\sqrt{0.15}} = \frac{-2}{1.08} = -1.85$$

State the results

$$Z = -1.85$$

$$Fail\ to\ reject\ H_0\ because\ -1.96 \le Z \le 1.96$$

Make the conclusion

The weight of the men in the city does not differ significantly from the weights of men in the country.

Two sample z-test with equal variance

It is used for testing the claims when data is collected on two samples with equal variance. For instance, let us consider a case where the weights of 60 men each from two different cities A and B in a country are normally distributed with means 65kg and 70kg respectively with equal standard deviation of 6. A researcher can test whether the weights of men in the two cities differ.

The case above is handled as follows:

State the hypothesis

$$H_o: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

Choose the level of significance

$$Let \ \propto = 0.05 \ or \ 5\%$$

State the decision rule

$$Reject\ H_0\ if\ Z < -1.96\ or\ Z > 1.96$$

$$Fail\ to\ reject\ H_0\ if\ -1.96 \leq Z \leq 1.96$$

Calculate the test statistics (Z-calculated)

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\delta\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$\bar{x}_1 = 65\ \bar{x}_2 = 70\ \delta = 6\ n_1 = 60\ n_1 = 60$$

$$Z = \frac{65 - 70}{6\sqrt{\frac{1}{60} + \frac{1}{60}}}$$

$$Z = \frac{-5}{6\sqrt{\frac{2}{60}}} = \frac{-5}{6\sqrt{0.03}} = \frac{-5}{6 \; x \; 0.18} = \frac{-5}{1.08} = -4.63$$

State the results

$$Z = -4.63$$

$$Reject \; H_0 \; because \; Z < -1.96$$

Make the conclusion

The weights of the men in the two cities are significantly different.

Two sample z-test with unequal variance.

Assuming a drink was manufactured and there is a claim that the drink increases weight when taken. Let us again assume that a sample size of 80 men of equal weights were selected in the country and 40 of them were given the drink and the other forty not. After six months weights of the men given the drink and those not given were determined and the following results obtained: Those given the drinks had a mean weight of 45kg with a standard deviation of 4 while those not given had a mean weight of 35kg with a standard deviation of 5. One can test whether the drink caused any significant change in the weights of men given the drink and those not. This can be tested below.

State the hypothesis

$$H_o: \bar{x}_1 = \bar{x}_2$$

$$H_o: \bar{x}_1 \neq \bar{x}_2$$

Choose the level of significance

State the decision rule

$$Reject\ H_0\ if\ Z < -1.96\ or\ Z > 1.96$$

$$Fail\ to\ reject\ H_0\ if\ -1.96 \leq Z \leq 1.96$$

Calculate the test statistics (Z- calculated)

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\delta_1^2}{n_1} + \frac{\delta_2^2}{n_2}}}$$

$$\bar{x}_1 = 45\ \bar{x}_2 = 35\ \delta_1^2 = 4\ \delta_2^2 = 5\ n_1 = 40\ n_1 = 40$$

$$Z = \frac{45 - 35}{\sqrt{\frac{4}{40} + \frac{5}{40}}}$$

$$Z = \frac{10}{\sqrt{\frac{9}{40}}} = \frac{10}{0.47} = 21.08$$

State the results

$$Z = 21.08$$

$$Reject\ the\ null\ hypothesis\ (H_0)\ because\ Z > 1.9$$

Make the conclusion

It can therefore be concluded that weights of men given the drink are significantly different from those not given. Hence the drink might have caused this difference in weight.

Chi-square Test

The chi-square test is used to test whether a relationship exist or not between two categorical variables; and also establish whether a number of outcomes are occurring with equal frequencies or not; or conforming to a known distribution or not.

Basically the chi-square is used for the following: test for hypothesized ratios; test for homogeneity of data collected from experimental trials that can be repeated; test for the independence of two groups attribute data – in other words it implies testing for whether they have a relationship.

The formula below is used in computing the chi-square (x^2) calculated value:

$$x^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

$$x^2 = Chi - square\ value, O = observed\ value\ E = expected\ value$$

Testing whether two outcomes occur with equal frequencies or not

Taking for instance a scenario where an individual is blindfolded and allowed to pick randomly each time a medal among five different medals placed on a table for 100 times, the data below is obtained:

Table 8.4.1 *Data used for Illustration of Chi-square Test.*

Medal type	Observed frequency selected medals
Diamond	25
Gold	30
Bronze	10
Silver	20
Aluminium	15

Now with this data, a researcher can test whether the frequency of the selection of medals are equal or not or follow a particular distribution using a chi-square test. In order to use a chi-square test, two particular set of data is required: the observed values and expected values. The observed values are those collected from the experiment or the study. The expected values are based on calculations.

For instance in the case at hand, it can be said that once the section was done 100 times and each of the medals given equal chances of being selected, it presupposes that the expected frequencies of each medal of being selected is 100. Thus a table below can be developed for the data:

Table 8.4.2 *Determining the Expected frequency from a given Data for calculating the Chi-square Test.*

Medal type	Observed frequency selected medals	Expected frequency of selected medals
Diamond	25	100
Gold	30	100
Bronze	10	100
Silver	20	100
Aluminium	15	100

The chi-square test for the data is computed as follows:

$$x^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

$$x^2 = \sum_{i=1}^{5} \frac{(O_d - E_d)^2}{E_d} + \frac{(O_g - E_g)^2}{E_g} + \frac{(O_b - E_b)^2}{E_b} + \frac{(O_s - E_s)^2}{E_s} + \frac{(O_a - E_a)^2}{E_a}$$

$$x^2 = \sum_{i=1}^{5} \frac{(25 - 100)^2}{100} + \frac{(30 - 100)^2}{100} + \frac{(10 - 100)^2}{100} + \frac{(20 - 100)^2}{100} + \frac{(15 - 100)^2}{100}$$

$$x^2 = \sum_{i=1}^{5} \frac{(-75)^2}{100} + \frac{(-70)^2}{100} + \frac{(-90)^2}{100} + \frac{(-80)^2}{100} + \frac{(-85)^2}{100}$$

$$x^2 = \sum_{i=1}^{5} \frac{5625}{100} + \frac{4900}{100} + \frac{8100}{100} + \frac{6400}{100} + \frac{7225}{100}$$

$$x^2 = \sum_{i=1}^{5} 56.25 + 49 + 81 + 64 + 72.25 = 322.5$$

Thus $x^2_{calculated} = 322.5$

$$df = n - 1 = 5 - 1 = 4$$

Therefore if $x^2_{calculated} > x^2_{critical}\ (df = 4, \alpha = 0.05\ 0r\ 0.01)$, then the frequencies of selecting the medals are significantly different.

However if $x^2_{calculated} < x^2_{critical}\ (df = 4, \alpha = 0.05\ or\ 0.01)$, then the frequencies of selecting the medals are not significantly different or equal.

Before a decision can be taken, the chi-square critical value must be read from the figure as show below:

The areas given across the top are the areas to the right of the critical value. To look up an area on the left, subtract it from one, and then look it up (ie: 0.05 on the left is 0.95 on the right)

df	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	---	---	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548

Figure 8.4.2 *Chi-square Test Table.*

From the table

$$x^2_{critical}\ (df = 4, \alpha = 0.05) = 9.488;$$

$$x^2_{critical}\ (df = 4, \alpha = 0.01) = 13.277$$

Decision:

$$x^2_{calculated}(> x^2_{critical}\ at\ df = 4, [\alpha = 0.01\ (13.28)]and\ [\alpha = 0.05\ (9.49)]$$

Conclusion:

The frequencies of selecting the medals are significantly different or are not equal.

Test for hypothesized ratios.

This test is used when there is an existing hypothesized ratio that is known to be working and a researcher has carried out a study relating to the ratio, he or she then can use chi-square to test whether the data collected in the study confirms or not the hypothesized ratio. Assuming there is known hypothesized ratio from a study that indicates that the number of birth per animal is in the ratio of 1:2 for male and female offspring respectively. If a researcher conducts an experiment on the births per animal over a period and recorded 92 and 212 male and female progenies respectively, then the chi-square test can be used to test this data collected against the hypothesized ratio.

In order to compute the chi-square value for this data, the expected values for each of the observation must be calculated.

Since the total birth per animal over the period is 92 + 212 = 304, the expected values for each of the birth per gender for the animal over the period can be calculated by using the hypothesized ratio i.e. 1:2. Thus the expected values for the birth per gender for the animal over the period are calculated as follow:

$$E_m = \frac{Ratio\ value\ for\ Male}{Total\ Ratio}\ x\ Total\ Observed\ Value$$

$$E_m = \frac{1}{3}\ x\ 304 = 101.33$$

$$E_m = \frac{Ratio\ value\ for\ Female}{Total\ Ratio}\ x\ Total\ Observed\ Value$$

$$E_f = \frac{2}{3}\ x\ 304 = 202.67$$

Table 8.5.1 *Data used as Illustration for testing of hypothesized ratio using Chi-square Test.*

	Birth per animal	
Gender	**Observed Values**	**Expected Values**
Male	92	101.33
Female	212	202.67
Total	304	304

$$x^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

$$x^2 = \sum_{i=1}^{2} \frac{(O_m - E_m)^2}{E_m} + \frac{(O_f - E_f)^2}{E_f}$$

$$x^2 = \sum_{i=1}^{5} \frac{(92 - 101.33)^2}{101.33} + \frac{(212 - 202.67)^2}{202.67}$$

$$x^2 = \sum_{i=1}^{5} \frac{87.05}{101.33} + \frac{87.05}{202.67} = 0.86 + 0.43 = 1.29$$

$$x^2 = 1.29$$

$$df = n - 1 = 2 - 1 = 1$$

Therefore if $x^2_{calculated} > x^2_{critical}$ $(df = 1, \alpha = 0.05 \; or \; 0.01)$ then the birth per gender animal is significantly different from the hypothesized ratio, that is to say the data collected does not confirm the hypothesized ratio.

However if $x^2_{calculated} < x^2_{critical}$ $(df = 1, \alpha = 0.05 \; or \; 0.01)$, then the birth per gender animal is not significantly different from the hypothesized ratio, that is to say that the data collected confirms the hypothesized ratio.

The areas given across the top are the areas to the right of the critical value. To look up an area on the left, subtract it from one, and then look it up (ie: 0.05 on the left is 0.95 on the right)

df	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	---	---	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757

Figure 8.5.1 *Chi-square Test Table.*

The

$$x^2_{critical}(\,at\ df\ = 1, \alpha = 0.05\,) = \ 3.841;$$

$$x^2_{critical}(\,at\ df\ = 1, \alpha = 0.01\,) = 6.635$$

$$However\ if\ x^2_{calculated} < x^2_{critical}\ (\,df\ = 1, \alpha = 0.05\ or\ 0.01),$$

Decision:

$$\mathrm{x}^2_{\mathrm{calculated}}\ = 1.29 > x^2_{\mathrm{critical}}(\,\mathrm{at\ df} = 1, \alpha = 0.05\,) = \ 3.841$$

$$\mathrm{x}^2_{\mathrm{calculated}}\ = 1.29 > x^2_{\mathrm{critical}}(\,\mathrm{at\ df} = 1, \alpha = 0.01\,) = 6.635$$

Conclusion:

The birth per gender animal is significantly different from the hypothesized ratio, that is to say the data collected does not confirm the hypothesized ratio.

Test for homogeneity of data collected from experimental trials that can be repeated.

Once the researcher is able to confirm that the collected data from a study follows the given hypothesized ratio and the study can be repeated, the researcher can test the homogeneity of the data collected on repeated trials for the study by the chi-square test. Thus the homogeneity of the data below can be tested by Chi-square test.

Table 8.5.2 *Data for Illustration of the Test of Homogeneity using Chi-square Test.*

Births per animal per time	Male	Female	Row Total
1st	12	26	38
2nd	9	16	25
3rd	15	27	42
4th	17	33	50
Column Total	53	102	155
Grand Total			155

In this situation, the researcher can compute the expected values by using the formula below:

$$E_V = \frac{R_T \ x \ C_T}{G_T}$$

E_V=Expected value, R_T=Row Total, C_T=Column Total, G_T=Grand Total.

Thus expected values for the 1st Trial:

Table 8.5.3 *Determining the Expected Values for the Data used for Test of Homogeneity.*

Births per animal per time	Male		Female		Row Total
1st	12	$E_V = \frac{38 \ x \ 53}{155}$	26	$E_V = \frac{38 \ x \ 102}{155}$	38
2nd	9	$E_V = \frac{25 \ x \ 53}{155}$	16	$E_V = \frac{25 \ x \ 102}{155}$	25
3rd	15	$E_V = \frac{42 \ x \ 53}{155}$	27	$E_V = \frac{42 \ x \ 102}{155}$	42
4th	17	$E_V = \frac{50 \ x \ 53}{155}$	33	$E_V = \frac{50 \ x \ 102}{155}$	50
Column Total	53		102		155
Grand Total					155

Table 8.5.4 *Determined Expected Values for the Data used for the Test of Homogeneity.*

Births per animal per time	Male Observed values	Male Expected values	Female Observed Values	Female Expected values	Row Total
1st	12	12.99	26	25.01	38
2nd	9	8.54	16	16.45	25
3rd	15	14.36	27	27.64	42
4th	17	17.10	33	32.90	50
Column Totals	53		102		155
Grand Total					155

The chi-square value for the data can now be computed as follows:

$$x^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

$$x^2 = \sum_{i=1}^{n} \frac{(12 - 12.99)^2}{12.99} + \cdots + \frac{(17 - 17.10)^2}{17.10} + \frac{(26 - 25.01)^2}{25.01} + \cdots + \frac{(33 - 32.90)^2}{32.90}$$

$$x^2 = \sum_{i=1}^{n} \frac{(-0.994)^2}{12.99} + \cdots + \frac{(-0.097)^2}{17.10} + \frac{(0.994)^2}{25.01} + \cdots + \frac{(0.097)^2}{32.90}$$

$$x^2 = \sum_{i=1}^{n} \frac{0.987}{12.99} + \cdots + \frac{0.009}{17.10} + \frac{0.987}{25.01} + \cdots + \frac{0.009}{32.90}$$

$$x^2 = \sum_{i=1}^{n} 0.076 + \cdots + 0.001 + 0.040 + \cdots + 0.0003 = 0.20$$

$$df = n - 1 = 4 - 1 = 3$$

The areas given across the top are the areas to the right of the critical value. To look up an area on the left, subtract it from one, and then look it up (ie: 0.05 on the left is 0.95 on the right)

df	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	---	---	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	~~0.072~~	~~0.115~~	~~0.216~~	~~0.352~~	~~0.584~~	~~6.251~~	7.815	~~9.348~~	11.345	12.838

Figure 8.5.2 *Chi-square Test Table.*

The read critical values from the table at both 1% and 5% levels of significance are:

$$x^2_{critical}(\,at\ df\ = 1, \alpha\ = 0.05\,) = 7.82$$

$$x^2_{critical}(\,at\ df\ = 1, \alpha\ = 0.01\,) = 11.35$$

Decision:

$$x^2_{calculated}\ = 0.20 < x^2_{critical}(\,at\ df\ = 1, \alpha\ = 0.05\,) = 7.82$$

$$x^2_{calculated}\ = 0.20 < x^2_{critical}(\,at\ df\ = 1, \alpha\ = 0.01\,) = 11.35$$

Conclusion:

It means the homogeneity of the data does not differ significantly for each trial. The data collected is similar for each trial.

Test for the independence of two groups attribute data

Considering another case where a researcher needs to establish whether a relationship exist between three different modes of admission of students into different programmes and the performance of the students. A data below can be subjected to the analysis:

Table 8.6.1 *Data used for Illustration of the Test for the Independence of Two Groups.*

Modes of students admission	**Grade Points of students in each Programme**		**Row Total**
	Mechanical Engineering	**Electrical Engineering**	
Regular	4.2	3.1	7.3
Access	3.2	3.4	6.6
Matured	3.6	2.1	5.7
Column Total	11	8.6	19.6

For the above data, the expected values for each observed value are computed as follows:

$$E_V = \frac{R_T \ x \ C_T}{G_T}$$

E_V=Expected value, R_T=Row Total, C_T=Column Total, G_T=Grand Total.

Table 8.6.2 *Determining the Expected Values for Data used for the Illustration of the Test for the Independence of Two Groups.*

Modes of students admission	**Grade Points of students in each Programme**				**Row Total**
	Mechanical Engineering		Electrical Engineering		
	Observed values	Expected values	Observed values	Expected Values	
Regular	4.2	$E_V = \frac{7.3 \ x \ 11}{19.6}$	3.1	$E_V = \frac{7.3 \ x \ 8.6}{19.6}$	7.3
Access	3.2	$E_V = \frac{6.6 \ x \ 11}{19.6}$	3.4	$E_V = \frac{6.6 \ x \ 8.6}{19.6}$	6.6
Matured	3.6	$E_V = \frac{5.7 \ x \ 11}{19.6}$	2.1	$E_V = \frac{5.7 \ x \ 8.6}{19.6}$	5.7
Column Total	11		8.6		19.6

Table 8.6.3 *Determined Expected Values for Data used for the Illustration of the Test for the Independence of Two Groups.*

Modes of students admission	**Grade Points of students in each Programme**				**Row Total**
	Mechanical Engineering		Electrical Engineering		
	Observed values	Expected values	Observed values	Expected Values	
Regular	4.2	4.1	3.1	3.2	7.3
Access	3.2	3.7	3.4	2.9	6.6
Matured	3.6	3.2	2.1	2.5	5.7
Column Total	11		8.6		19.6

The chi-square value for the data can now be computed as follows:

$$x^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

$$x^2 = \sum_{i=1}^{n} \frac{(4.2 - 4.1)^2}{12.99} + \cdots + \frac{(3.6 - 3.2)^2}{17.10} + \frac{(3.1 - 3.2)^2}{25.01} + \cdots + \frac{(2.1 - 2.5)^2}{32.90}$$

$$x^2 = \sum_{i=1}^{3} \frac{(0.1)^2}{12.99} + \cdots + \frac{(0.4)^2}{17.10} + \frac{(-0.1)^2}{25.01} + \cdots + \frac{(-0.4)^2}{32.90}$$

$$x^2 = \sum_{i=1}^{3} \frac{(0.1)^2}{12.99} + \cdots + \frac{(0.4)^2}{17.10} + \frac{(-0.1)^2}{25.01} + \cdots + \frac{(-0.4)^2}{32.90}$$

$$x^2 = \sum_{i=1}^{3} \frac{(0.1)^2}{12.99} + \cdots + \frac{(0.4)^2}{17.10} + \frac{(-0.1)^2}{25.01} + \cdots + \frac{(-0.4)^2}{32.90}$$

$$x^2 = \sum_{i=1}^{3} \frac{0.01}{12.99} + \cdots + \frac{0.16}{17.10} + \frac{0.01}{25.01} + \cdots + \frac{0.16}{32.90}$$

$$x^2 = \sum_{i=1}^{3} 0.002 + \cdots + 0.050 + 0.003 + \cdots + 0.020 = 0.23$$

$$x^2 = 0.23$$

$$df = (n_1 - 1)(n_2 - 1) = (3-1)(2-1) = 3$$

The areas given across the top are the areas to the right of the critical value. To look up an area on the left, subtract it from one, and then look it up (ie: 0.05 on the left is 0.95 on the right)

df	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	---	---	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	~~0.072~~	~~0.115~~	~~0.216~~	~~0.352~~	~~0.584~~	~~6.251~~	7.815	~~9.348~~	11.345	12.838

Figure 8.6.1 *Chi-square Test Table.*

The read critical values from the table at both 1% and 5% levels of significance are:

$$x^2_{critical}(\, at\ df = 1, \alpha = 0.05\,) = 7.82$$

$$x^2_{critical}(\, at\ df = 1, \alpha = 0.01\,) = 11.35$$

It should be noted that with the testing of the independency of two variables or groups data attribute. These hypotheses are put forward:

$$H_0: the\ two\ groups\ are\ independent\ of\ eacher\ other$$

$$H_1: one\ variable\ or\ group\ depend\ on\ the\ other$$

Decision:

$$x^2_{calculated} = 0.23 < x^2_{critical}(\, at\ df = 1, \alpha = 0.05\,) = 7.82$$

$$x^2_{calculated} = 0.23 < x^2_{critical}(\ at\ df = 1, \alpha = 0.01\ ) = 11.35$$

Conclusion:

$$since\ x^2_{calculated}(0.23) < x^2_{critical}(7.82)\ at\ df = 1,$$
$$\alpha = 0.0\ and\ (11.35)\ at\ df = 3, \alpha = 0.01$$

we fail to reject the null hypothesis.

It means the data supports the null hypothesis which is the mode of admission is independent of the performance of the students. The data collected is similar for each trial.

Analysis of Variance (ANOVA)

Analysis of Variance has the acronym (ANOVA). It is used to test whether there is a difference between the means of two or more continuous dependent variables or to test the difference in the means of a single continuous dependent variable determined at two or more different times. Whereas the t-test employs the t-statistics, ANOVA employs the F-test or the F-statistics. When finding the difference between two groups, both the t-statistics and the F-statistics give the same results. However ANOVA can be used to test whether or not differences exist between two or more groups whereas t-test can be used to test whether there is a difference in the means of just two groups at a time.

Assuming a researcher collects data on the performances in terms of the final grade points obtained by students enrolled in mechanical, electrical and civil engineering at the end of their programmes and summarized it in a table 8.7.1.

Table 8.7.1 *Data for the Illustration of ANOVA.*

Grade Points of students in each Programme		
Mechanical Engineering (m)	Civil Engineering (c)	Electrical Engineering (e)
4.2	3.2	3.1
3.2	2.1	3.4
3.6	4.1	2.1
3.7	2.3	4.4
2.3	4.2	3.7

The researcher can test whether means of the performance of the students in the different programmes differ or are the same using ANOVA.

To use the data in the table above to test whether the mean performances of the students differ between the programmes, the researcher must do the calculations below.

After the data have been collected the researcher needs to calculate the following from the data:

- Sum of Scores of each group = $\sum x$.
- Mean of Scores of each group = $\bar{x}$.
- Sum of the Squares of Scores of each group = $\sum x^2$.
- Summation of the Sums of the Scores for all the groups = $\sum(\sum x)$.
- Summation of the Sums of squares of the Scores for all the groups = $\sum(\sum x^2)$.

Let us proceed to calculate these variables for the data:

First, we calculate the sums of the scores, the sums of squares and the means of each group

$$\sum x_m = m_1 + m_2 + m_3 \dots \dots . + m_n$$

$$\sum x_m = 4.2 + 3.2 + 3.6 \dots + 2.3 = 17$$

$$\bar{x}_m = \frac{\sum x_m}{n_m} = \frac{17}{5} = 3.40$$

$$\sum x_c = c_1 + c_2 + c_3 \dots \dots . + c_n$$

$$\sum x_c = 3.2 + 2.1 + 4.1 \dots + 4.2 = 15.9$$

$$\bar{x}_c = \frac{\sum x_c}{n_c} = \frac{15.9}{5} = 3.18$$

$$\sum x_e = e_1 + e_2 + e_1 \dots \dots . + e_n$$

$$\sum x_e = 3.1 + 3.4 + 2.1 \dots + 3.7 = 16.7$$

$$\bar{x}_e = \frac{\sum x_e}{n_e} = \frac{16.7}{5} = 3.34$$

$$\sum {x_m}^2 = {m_1}^2 + {m_2}^2 + {m_3}^2 \dots + {m_n}^2$$

$$\sum {x_m}^2 = 4.2^2 + 3.2^2 + 3.6^2 \dots + 2.3^2 = 59.82$$

$$\sum {x_c}^2 = {c_1}^2 + {c_2}^2 + {c_3}^2 \dots + {c_n}^2$$

$$\sum {x_c}^2 = 3.2^2 + 2.1^2 + 4.1^2 \dots + 4.2^2 = 54.39$$

$$\sum {x_e}^2 = {e_1}^2 + {e_2}^2 + {e_3}^2 \dots + {e_n}^2$$

$$\sum {x_e}^2 = 4.2^2 + 3.2^2 + 3.6^2 \dots + 2.3^2 = 58.63$$

The next step is to find the summation of the sums of scores of each group; and the summation of the sums of squares of scores for each group:

$$\sum (\sum x) = \sum x_m + \sum x_c + \sum x_e = 17 + 15.9 + 16.7 = 49.6$$

$$\sum(\sum x^2) = \sum x_m{}^2 + \sum x_c{}^2 + \sum x_e{}^2 \; = 59.82 + 54.39 + 58.63 = 172.84$$

Calculate for the Total Sum of Squares; Sum of Squares between groups; and the Sum of Squares within groups

$$SS_T = \sum\left(\sum x^2\right) - \frac{(\sum(\sum x)^2}{n_T}$$

$$SS_T = \sum\left(\sum x^2\right) - CF$$

$$CF = \frac{(\sum(\sum x)^2}{n_T}$$

$$SS_T = \sum 172.84 - \frac{(49.6)^2}{15}$$

$$SS_T = \sum 172.84 - \frac{2460.16}{15} = 172.84 - 164.01 = 8.83$$

CF=Correction factorm, n_T=n_m+n_c+n_e =Total number of scores, SS_T=Total Sum of Squares n=n_e=n_e=n_e=number scores for each group

$$SS_B = \sum\frac{(\sum x)^2}{n} - \frac{(\sum(\sum x)^2}{n_T}$$

$$SS_B = \sum(\frac{(x_m)^2}{n_m} + \frac{(x_c)^2}{n_c} + \frac{(x_e)^2}{n_e}) - \frac{(\sum(\sum x)^2}{n_T}$$

$$SS_B = \sum\left(\frac{(17)^2}{5} + \frac{(15.9)^2}{5} + \frac{(16.7)^2}{5}\right) - \frac{(49.6)^2}{15}$$

$$SS_B = \sum\left(\frac{289}{5} + \frac{252.81}{5} + \frac{278.89}{5}\right) - \frac{2460.16}{15}$$

$$SS_B = \sum(57.8 + 50.56 + 55.78) - \; 164.01 = 164.14 - 164.01 = 0.13$$

SS_B=Sum of Squares between the groups, n=n_m=n_c=n_e=number scores for each group

$$SS_T = SS_B + SS_W$$

Therefore

$$SS_W = SS_T - SS_B$$

$$SS_W = 8.83 - 0.13 = 8.7$$

Where SS_W=Sum of Squares within the groups

$$df_T = df_B + df_W$$

$$df_T = n - 1 = 15 - 1 = 14$$

$$df_B = n - 1 = 3 - 1 = 2 \;\; df_W = N_T - \; n = 15 - 3 = 12$$

The Means Squares of between and within groups are then computed:

$$MS_B = \frac{SS_B}{df_B}$$

$$MS_B = \frac{0.13}{2} = 0.07$$

$$MS_W = \frac{SS_W}{df_W}$$

$$MS_W = \frac{8.7}{12} = 0.73$$

where MS_B=Mean Square between groups; MS_W=Mean Squares within groups; df_B=between groups degree of freedom; df_W=within groups degree of freedom.

Finally compute the F-calculated value and read F-critical values from F-test table for comparison:

$$F = \frac{MS_B}{MS_W}$$

$$F_{calculated} = \frac{0.07}{0.73} = 0.10$$

Reading of F-critical values at 5% and 1% levels of significance shown below:

Critical values of F for the 0.05 significance level:

	1	2	3	4	5	6
1	161.45	199.50	215.71	224.58	230.16	233.99
2	18.51	19.00	19.16	19.25	19.30	19.33
3	10.13	9.55	9.28	9.12	9.01	8.94
4	7.71	6.94	6.59	6.39	6.26	6.16
5	6.61	5.79	5.41	5.19	5.05	4.95
6	5.99	5.14	4.76	4.53	4.39	4.28
7	5.59	4.74	4.35	4.12	3.97	3.87
8	5.32	4.46	4.07	3.84	3.69	3.58
9	5.12	4.26	3.86	3.63	3.48	3.37
10	4.97	4.10	3.71	3.48	3.33	3.22
11	4.84	3.98	3.59	3.36	3.20	3.10
12	4.75	3.89	3.49	3.26	3.11	3.00
13	4.67	3.81	3.41	3.18	3.03	2.92
14	4.60	3.74	3.34	3.11	2.96	2.85
15	4.54	3.68	3.29	3.06	2.90	2.79

Critical values of F for the 0.01 significance level:

	1	2	3	4	5	6
1	4052.19	4999.52	5403.34	5624.62	5763.65	5858.97
2	98.50	99.00	99.17	99.25	99.30	99.33
3	34.12	30.82	29.46	28.71	28.24	27.91
4	21.20	18.00	16.69	15.98	15.52	15.21
5	16.26	13.27	12.06	11.39	10.97	10.67
6	13.75	10.93	9.78	9.15	8.75	8.47
7	12.25	9.55	8.45	7.85	7.46	7.19
8	11.26	8.65	7.59	7.01	6.63	6.37
9	10.56	8.02	6.99	6.42	6.06	5.80
10	10.04	7.56	6.55	5.99	5.64	5.39
11	9.65	7.21	6.22	5.67	5.32	5.07
12	9.33	6.93	5.95	5.41	5.06	4.82
13	9.07	6.70	5.74	5.21	4.86	4.62
14	8.86	6.52	5.56	5.04	4.70	4.46
15	8.68	6.36	5.42	4.89	4.56	4.32

Figure 8.7.1 *F-critical Table.*

In order to read the F-critical values at the respective levels of significance, one must use the between groups degree of freedom along the row (horizontal) and then the within groups degree of freedom (df) along the column (vertical). The meeting points of these degrees of freedom on the table lies the F-critical values respectively for each chosen level of significance table.

Thus from the tables, the following are read as the F-critical values for the case being dealt with:

$$F_{critical}\,(2, 14\ df and\ 5\%) = 3.74;\ and\ F_{critical}\,(2, 14\ df and\ 1\%) = 6.52$$

Now we can complete the ANOVA table:

Table 8.7.2 *Completed ANOVA Table.*

Sources of Variations	df	SS	MS	F-calc.	Fcrit at 5%	Fcrit at 1%
Between groups	2	0.13	0.07	0.10	3.74	6.52
Within groups	14	8.7	0.73			
Total	14	8.83				

Now the researcher can compare the F-calculated values with the F-critical values at both levels of significance to take the decision and make conclusions on the case.

It should however be noted that whenever F-calculated is greater than F-critical at stated degrees of freedom and a given level of significance (α), the p-value (p-value < α) would also be less than the chosen level of significance. The contrary also holds i.e. if F-calculated is lesser than F-critical, p-value (p-value >α) would be less than the chosen level of significance.

Decision:

if $F_{(Calculated\ )}> F_{(Critical)}$ at (5% or 1%, df=2, 14); it means the performances of the students in the various programmes are significantly different.

However, if $F_{(Calculated)}< F_{(Critical)}$ (at 5% or 1%, df=2, 14); it suggests that the performances of the students are not significantly different.

For the case understudy:

$$F_{Calculated}\,(0.10) < \,F_{Critical}\,(at\ 5\%, df\ = 2, 14) = 3.74$$

$$F_{Calculated}\,(0.10) < \,F_{Critical}\,(at\ 1\%, df\ = 2, 14) = 6.52$$

Conclusion:

It can therefore be concluded that there is enough evidence presented by the data collected from the study that the performances of the students in the respective programmes are not significantly different or in other words are the same at both levels of significance.

Table I: Chi-Square Probabilities

The areas given across the top are the areas to the right of the critical value. To look up an area on the left, subtract it from one, and then look it up (ie: 0.05 on the left is 0.95 on the right)

Df	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	---	---	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

Table II: Standard Normal (Z)

Area between 0 and z

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

Table III: Student's T Table

T Table With Right Tail Probabilities

df\p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	4.3178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
inf	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905

Table IV: Chi-Square Table

df\ area	0.995	0.99	0.975	0.95	0.9	0.75	0.5	0.25	0.1	0.05
1	0.00004	0.00016	0.00098	0.00393	0.01579	0.10153	0.45494	1.3233	2.70554	3.84146
2	0.01003	0.0201	0.05064	0.10259	0.21072	0.57536	1.38629	2.77259	4.60517	5.99146
3	0.07172	0.11483	0.2158	0.35185	0.58437	1.21253	2.36597	4.10834	6.25139	7.81473
4	0.20699	0.29711	0.48442	0.71072	1.06362	1.92256	3.35669	5.38527	7.77944	9.48773
5	0.41174	0.5543	0.83121	1.14548	1.61031	2.6746	4.35146	6.62568	9.23636	11.0705
6	0.67573	0.87209	1.23734	1.63538	2.20413	3.4546	5.34812	7.8408	10.64464	12.59159
7	0.98926	1.23904	1.68987	2.16735	2.83311	4.25485	6.34581	9.03715	12.01704	14.06714
8	1.34441	1.6465	2.17973	2.73264	3.48954	5.07064	7.34412	10.21885	13.36157	15.50731
9	1.73493	2.0879	2.70039	3.32511	4.16816	5.89883	8.34283	11.38875	14.68366	16.91898
10	2.15586	2.55821	3.24697	3.9403	4.86518	6.7372	9.34182	12.54886	15.98718	18.30704
11	2.60322	3.05348	3.81575	4.57481	5.57778	7.58414	10.341	13.70069	17.27501	19.67514
12	3.07382	3.57057	4.40379	5.22603	6.3038	8.43842	11.34032	14.8454	18.54935	21.02607
13	3.56503	4.10692	5.00875	5.89186	7.0415	9.29907	12.33976	15.98391	19.81193	22.36203
14	4.07467	4.66043	5.62873	6.57063	7.78953	10.16531	13.33927	17.11693	21.06414	23.68479
15	4.60092	5.22935	6.26214	7.26094	8.54676	11.03654	14.33886	18.24509	22.30713	24.99579
16	5.14221	5.81221	6.90766	7.96165	9.31224	11.91222	15.3385	19.36886	23.54183	26.29623
17	5.69722	6.40776	7.56419	8.67176	10.08519	12.79193	16.33818	20.48868	24.76904	27.58711
18	6.2648	7.01491	8.23075	9.39046	10.86494	13.67529	17.3379	21.60489	25.98942	28.8693
19	6.84397	7.63273	8.90652	10.11701	11.65091	14.562	18.33765	22.71781	27.20357	30.14353
20	7.43384	8.2604	9.59078	10.85081	12.44261	15.45177	19.33743	23.82769	28.41198	31.41043
21	8.03365	8.8972	10.2829	11.59131	13.2396	16.34438	20.33723	24.93478	29.61509	32.67057
22	8.64272	9.54249	10.98232	12.33801	14.04149	17.23962	21.33704	26.03927	30.81328	33.92444
23	9.26042	10.19572	11.68855	13.09051	14.84796	18.1373	22.33688	27.14134	32.0069	35.17246
24	9.88623	10.85636	12.40115	13.84843	15.65868	19.03725	23.33673	28.24115	33.19624	36.41503
25	10.51965	11.52398	13.11972	14.61141	16.47341	19.93934	24.33659	29.33885	34.38159	37.65248
26	11.16024	12.19815	13.8439	15.37916	17.29188	20.84343	25.33646	30.43457	35.56317	38.88514
27	11.80759	12.8785	14.57338	16.1514	18.1139	21.7494	26.33634	31.52841	36.74122	40.11327
28	12.46134	13.56471	15.30786	16.92788	18.93924	22.65716	27.33623	32.62049	37.91592	41.33714
29	13.12115	14.25645	16.04707	17.70837	19.76774	23.56659	28.33613	33.71091	39.08747	42.55697
30	13.78672	14.95346	16.79077	18.49266	20.59923	24.47761	29.33603	34.79974	40.25602	43.77297

Table V: F Distribution Tables

F - Table for alpha = 0.10

df2/df1	1	2	3	4	5	6	7	8	9	10	12	15	20
1	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86	60.19	60.71	61.22	61.74
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.41	9.42	9.44
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.20	5.18
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.90	3.87	3.84
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.27	3.24	3.21
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.90	2.87	2.84
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.67	2.63	2.59
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.50	2.46	2.42
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.38	2.34	2.30
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.28	2.24	2.20
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.21	2.17	2.12
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.15	2.10	2.06
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.10	2.05	2.01
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.05	2.01	1.96
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.02	1.97	1.92
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	1.99	1.94	1.89
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.96	1.91	1.86
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.93	1.89	1.84
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.91	1.86	1.81
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.89	1.84	1.79
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92	1.87	1.83	1.78
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.86	1.81	1.76
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89	1.84	1.80	1.74
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.83	1.78	1.73
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87	1.82	1.77	1.72
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.81	1.76	1.71
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85	1.80	1.75	1.70
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.79	1.74	1.69
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83	1.78	1.73	1.68
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.77	1.72	1.67
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.71	1.66	1.61

F -Table for alpha = 0.05

df2/df1	1	2	3	4	5	6	7	8	9	10	12	15
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.95
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75

F -Table for alpha = 0.025

df2/df1	1	2	3	4	5	6	7	8	9	10	12	15
1	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28	968.63	976.71	984.87
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.60	2.50
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.51	2.41
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.49	2.39
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.47	2.36
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.45	2.34
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.43	2.32
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.29	2.18
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.05	1.95
Inf	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83

F -Table for alpha = 0.01

df2/df1	1	2	3	4	5	6	7	8	9	10	12
1	4052.18	4999.50	5403.35	5624.58	5763.65	5858.99	5928.36	5981.07	6022.47	6055.85	6106.32
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89
6	13.75	10.93	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96
14	8.86	6.52	5.56	5.04	4.70	4.46	4.28	4.14	4.03	3.94	3.80
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.90	3.81	3.67
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55
17	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46
18	8.29	6.01	5.09	4.58	4.25	4.02	3.84	3.71	3.60	3.51	3.37
19	8.19	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12
23	7.88	5.66	4.77	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03
25	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.22	3.13	2.99
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96
27	7.68	5.49	4.60	4.11	3.79	3.56	3.39	3.26	3.15	3.06	2.93
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90
29	7.60	5.42	4.54	4.05	3.73	3.50	3.33	3.20	3.09	3.01	2.87
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.67
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34
inf	6.64	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.19

Table VI: Standard Normal (Z) Table

Values in the table represent areas under the curve to the left of Z quantiles along the margins.

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

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